UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Efficient Anti-community Detection in Complex Networks

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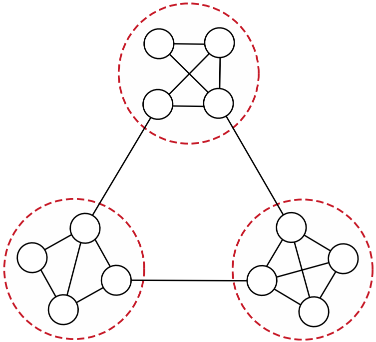
Community Structure

Many networks contain community structures.

Communities are characterized by

- many internal edges
- few external edges

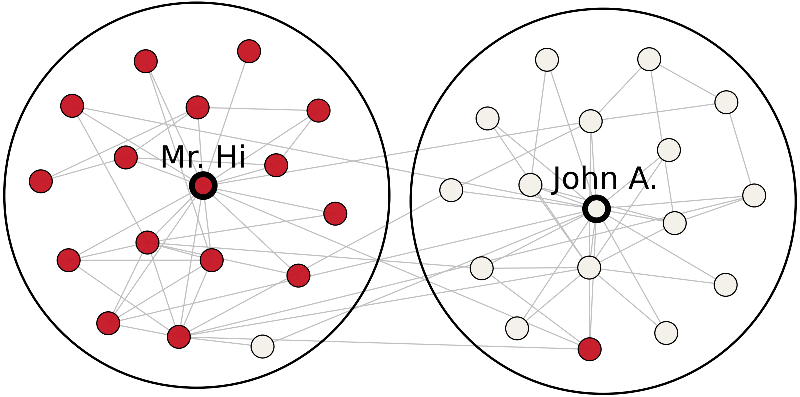
(*generalization of cliques*)



Applications in sociology, computer science, physics, biology, . . . [For10]

Zachary's Karate Club Network

$|V| = 34,$
$|E| = 156$



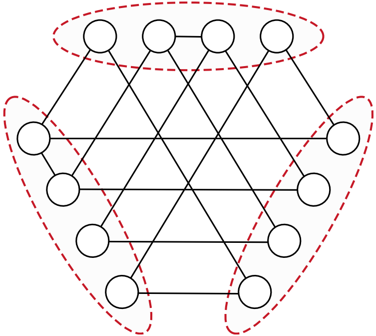
Communities in *Zachary's karate club* network [Zac77].
Colors denote membership after the fission of the club.

Anti-community Structure

Anti-Communities are characterized by

- few internal edges
- many external edges

  (*generalization of multipartite graphs*)



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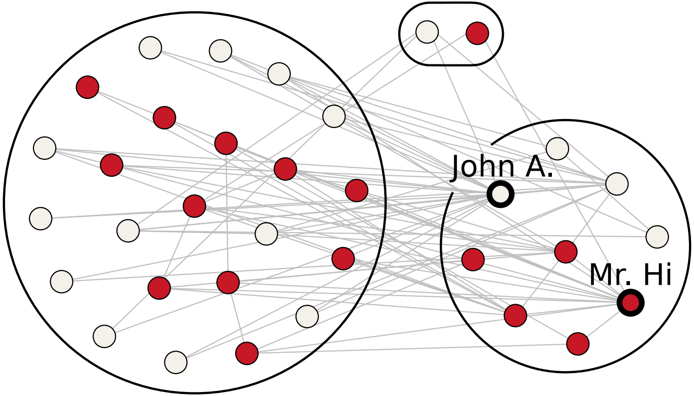

Anti-communities in *Zachary's karate club* network [Zac77].
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Challenges and Objectives

- **Definition**

  How to define anti-communities?

- **Models and Algorithms**

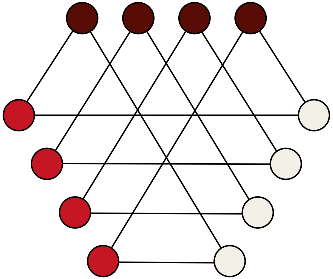
  Which algorithms can be used?

- **Exploratory Analysis**

  Are anti-communities also present in other networks?

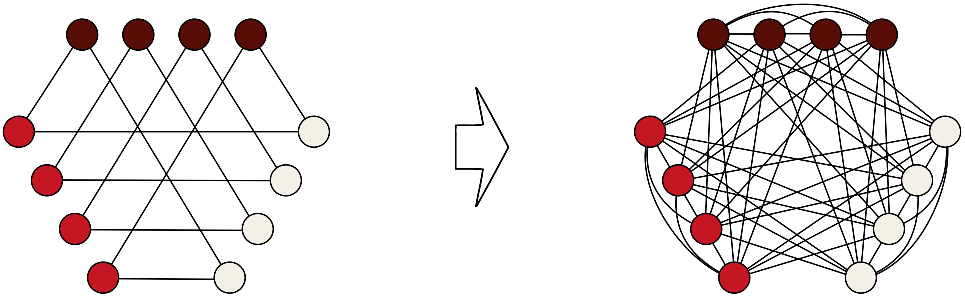
Definition

Graph Complement



Original network
with 3 anti-communities

Graph Complement



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Definition

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Vertices $C \subseteq V$ of graph $G = (V, E)$ form an anti-community iff C forms a community in the graph complement $\hat{G} = (V, \hat{E})$ with $\hat{E} := (V \times V) \setminus E$.

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Conclusions:

- Not really unique (many definitions for communities)
- Many existing algorithms and methods can be reused

Models and Algorithms

Proposed Methods

Existing methods either slow or poor quality.

Greedy algorithms

- using *Modularity* measure [NG04]
- using *Anti-Modularity* measure [CYC14]

Vertex similarity

- Adjacency mapping
- Distance mapping

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} Optimization problem

Vertex similarity

- Adjacency mapping
- Distance mapping

} Clustering problem

Modularity Measure

Intuition: Number of internal edges in $\mathcal{G} = (V, E)$ minus number of edges in a random graph with same degree-distribution.

Modularity of a graph

$$M := \frac{1}{2m} \sum_{ij} \left[a_{ij} - \frac{d_i d_j}{2m} \right] \delta(g_i, g_j)$$

m:	Total number of edges
$\mathbf{A} = [a_{ij}]$:	Adjacency matrix of $\mathcal{G}$
$d = [d_i]$:	Vertex degrees
$\delta(g_i, g_j)$:	1 iff v_i and v_j are both in same group

Greedy Algorithms

Make locally optimal choice at each step.

1. **Initialization**

   Assign each vertex to a separate group

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3. **Repeat**

   If more than one group is left, go to step 2.
   Otherwise, return groups with best *(Anti-)Modularity*.

Based on the concept of structural equivalence.

1. **Mapping**

   Map vertices to feature vector representation

   - **Adjacency mapping:** $M(v_i) := [a_{ij}]_j$
   - **Distance mapping:** $M(v_i) := [d(v_i, v_1), \ldots, d(v_i, v_n)]$

Vertex Similarity

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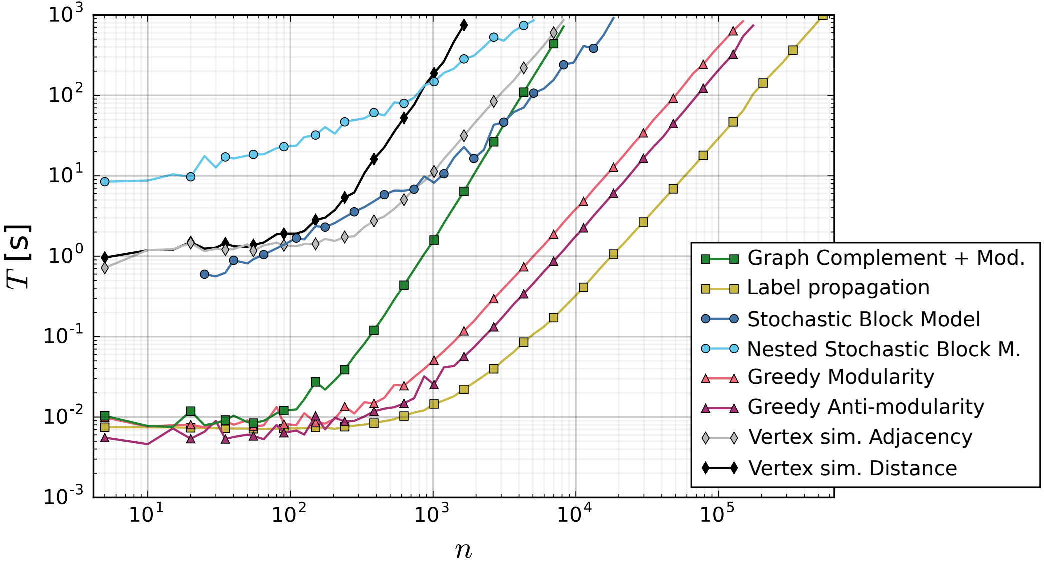
2. **Clustering**

   Compute clustering of feature vectors (*k-Means*, . . .)

Runtime Evaluation

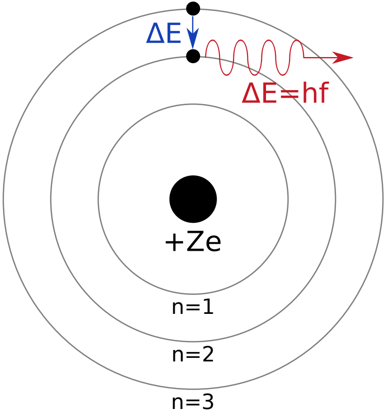


Evaluation with Erdős-Rényi random graphs (sparse)

Exploratory Analysis

Spectral Line Networks

Goal: Encode energy states of a physical system (and their relation) in a network.



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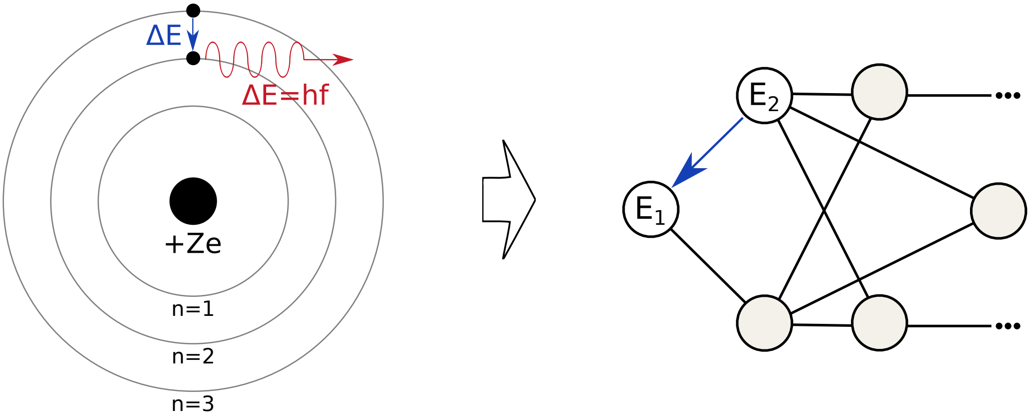


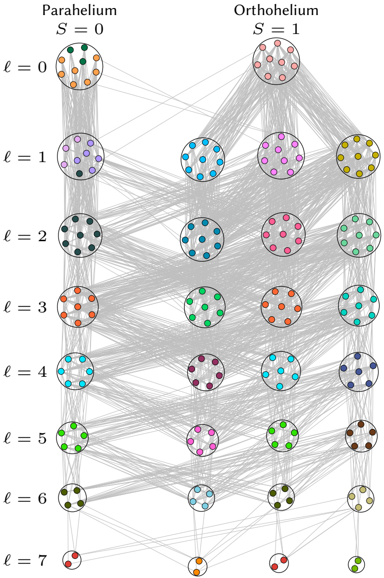
Example: *Spectral Line Network of Helium*

Spectral line network network of *Helium* [KRRN15] with $|V| = 183$, $|E| = 2282$.

Colors show the anti-communities obtained with a vertex similarity method.

Circles show the ground-truth partition

- orbital angular momentum (ℓ),
- total angular momentum (j), and
- spin (s)



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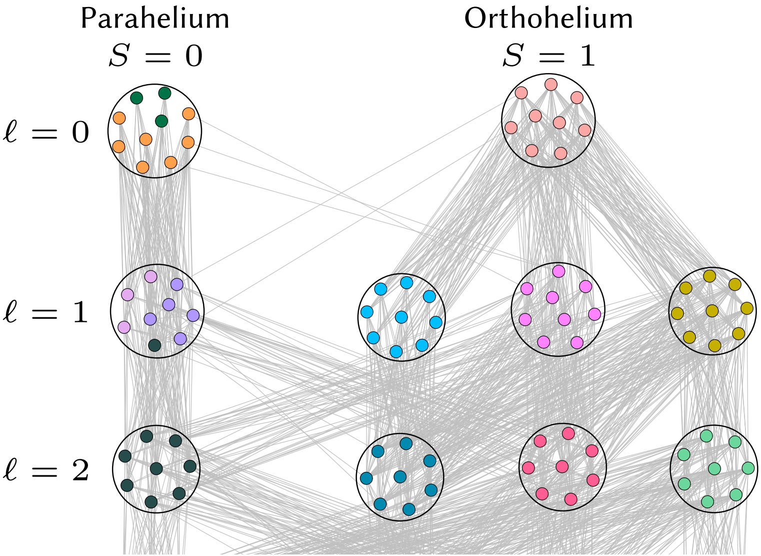
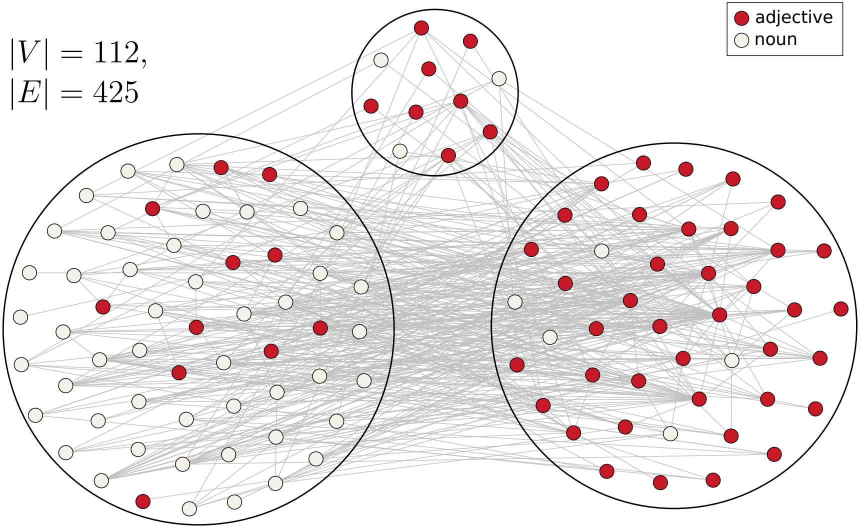


Example: *Adjectives and Nouns* Network

$|V| = 112,$
$|E| = 425$



Adjectives and Nouns network [New06]. Circles correspond to the anti-communities found by the greedy modularity minimization algorithm.

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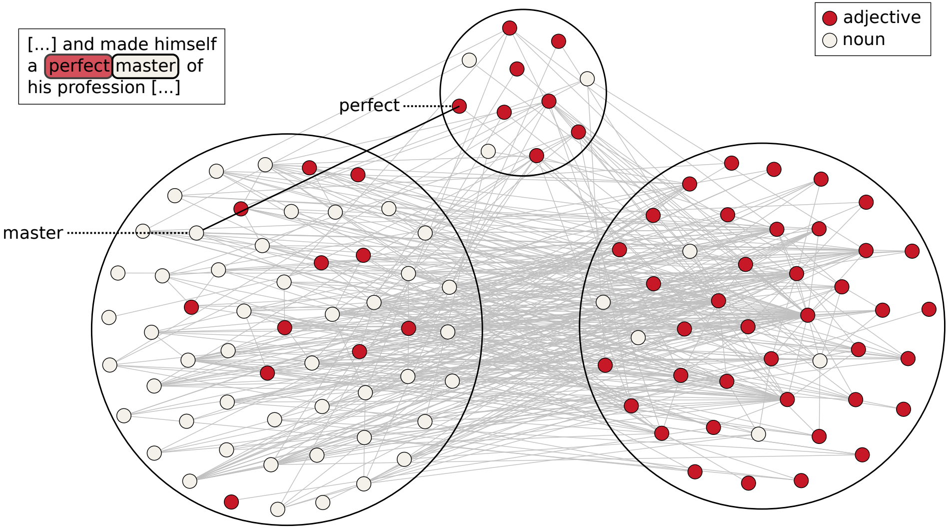

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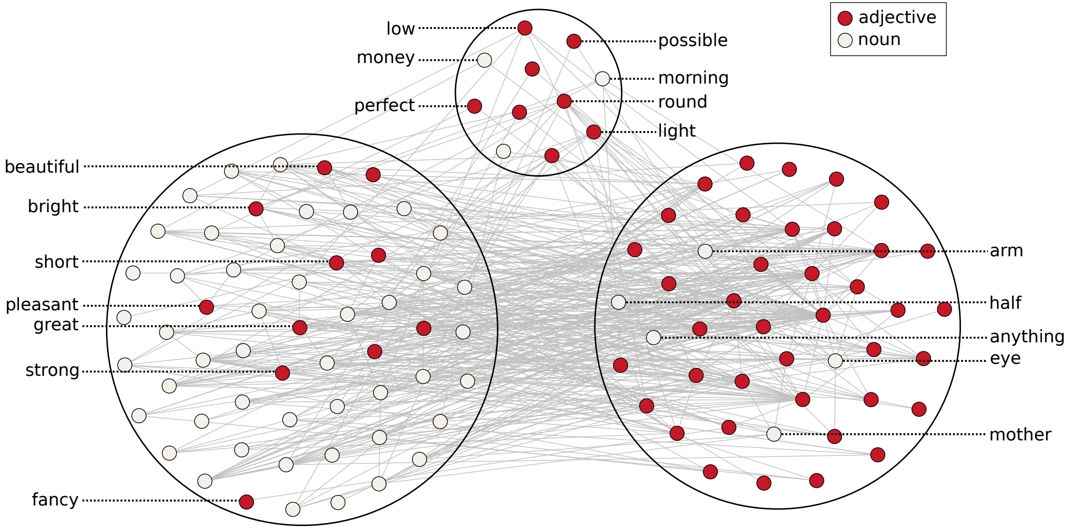

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Summary

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- Anti-community structures are present in many networks, including
  - networks of spectral line transitions
  - Zachary's karate club network
  - . . . and many more

- Many concepts of traditional community detection can be reused by computing the graph complement

- Specialized algorithms and measures are required if performance is important

Further Reading

- **Evaluation measures:**

  Adaption of the adjusted Rand index and normalized mutual information measures for anti-communities.

- **Random graphs:**

  Algorithms to generate Erdős-Rényi and Barabási-Albert random graph model for graphs with (anti-)community structure.

- **Performance evaluation:**

  Quality comparison for graphs with known community structure.

Resources

Implementations and datasets available at:

`http://dbs.ifi.uni-heidelberg.de/resources/anticommunity`

Thank you!

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Backup Slides

Baseline Methods

- **Graph complement + X**

  Allows to reuse existing methods, but high memory usage / slow.

- **Label propagation algorithm for anti-communities** [CYC14]

  Fast, but poor quality

- **Generic methods**

  e.g., Stochastic block models [Pei14; Pei17]

Complexity of Greedy Algorithms

- **Community detection:**

Naive method	$\mathcal{O}(n^3)$
Skip unconnected edges	$\mathcal{O}(n(n+m))$
Use max-heap data structure	$\mathcal{O}(n \log^2 n)$[1]

[1] for graphs with strong hierarchical structure

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Graph complement	$\mathcal{O}(n^3)$
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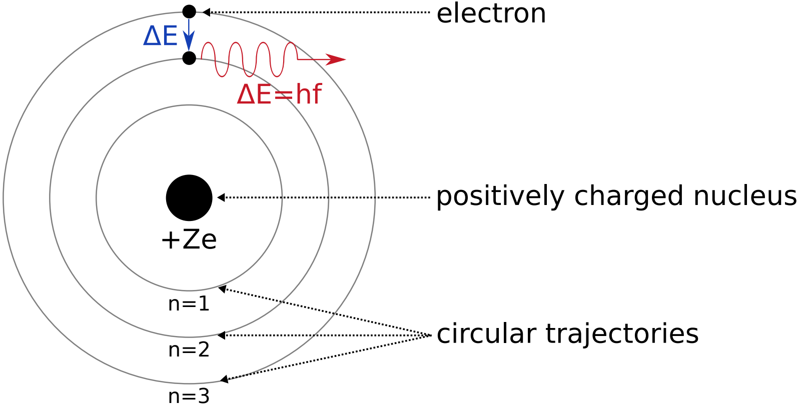
Result can also be used to improve community detection!

[1] for graphs with strong hierarchical structure

Basics of the *Bohr Model*

Goal: Encode energy states of a physical system (and their relation) in a network.



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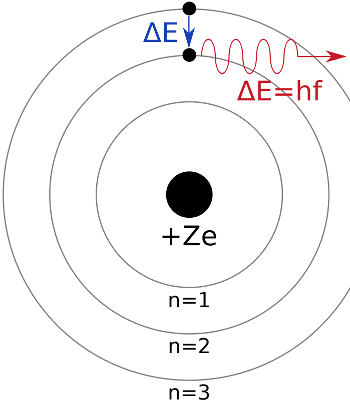

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  defined by possible orbits of electrons

- **State transitions**

  requires / releases energy ΔE

  $\rightarrow$ emission or absorption line

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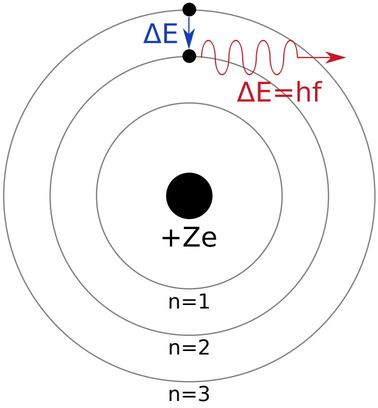

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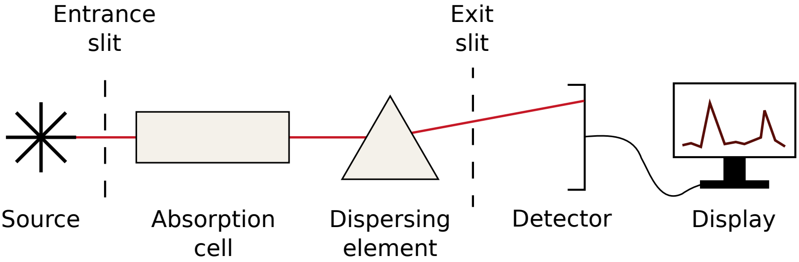
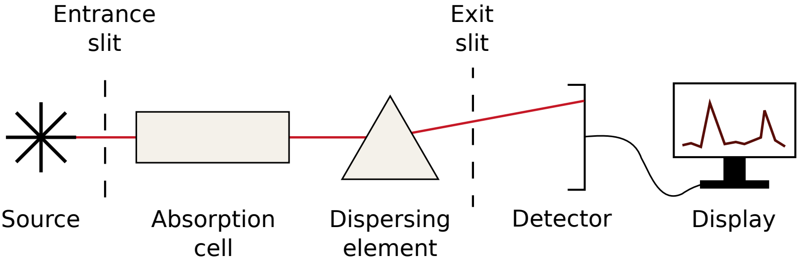
⚠ Simplified model!



Overview of an absorption experiment. Visualization based on *Modern Spectroscopy* by Hollas [Hol04].

Spectral Line Networks



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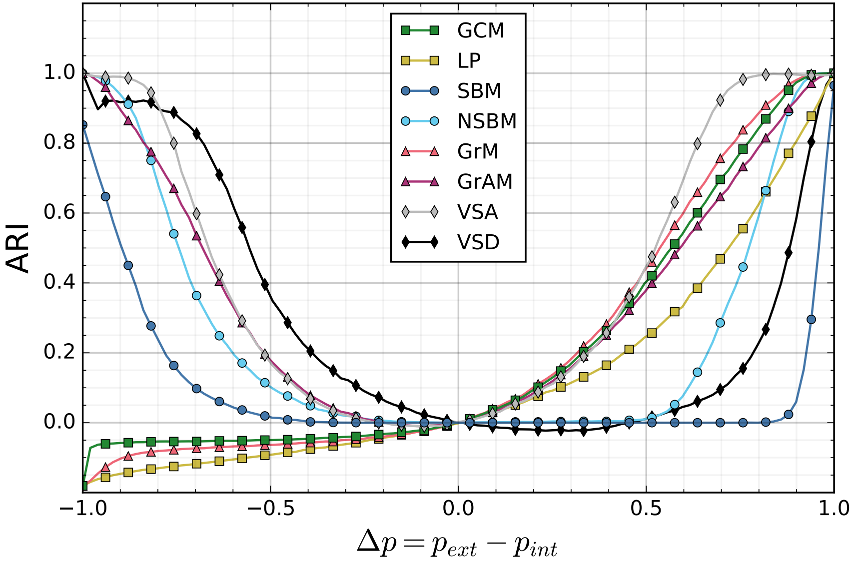
Spectral lines ▶ **State transitions** ▶ **Energy states** ▶ **Network**

Performance evaluation



Evaluation with Erdős-Rényi random graphs ($k = 5$)